Fuzzy path tracking control for automatic steering of vehicles

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Abstract

In this paper, the path tracking (PT) control for automatic steering of vehicles is studied. The Takagi–Sugeno (T–S) fuzzy model of vehicle obtained from a nonlinear model is considered and a fuzzy controller is designed. The stability analysis is discussed using Lyapunov's approach combined with the linear matrix inequalities (LMI) approach. Finally, simulation results are given to demonstrate the controller's effectiveness.
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Keywords: Path tracking; Fuzzy logic controller; Takagi–Sugeno fuzzy model; Vehicle dynamics

1. Introduction

Recently, fuzzy control has become a popular research in control engineering. The fuzzy logic controller has made itself available not only in the laboratory work but also in industrial applications [1–11]. In recent years, theoretical developments of fuzzy control have been proposed, and the constructions and the use of fuzzy controllers have been explored [12–18,20,22]. These works are essentially based on a fuzzy model of the process [19] and on Lyapunov stability to design the fuzzy control law.

One important application of fuzzy control is in vehicles: maritime, space and ground vehicles. In [1], Waneck proposed a fuzzy controller for an autonomous boat without initially having to develop a nonlinear dynamics model of a vehicle. Sugeno et al. [2,3] has designed a fuzzy controller based on fuzzy modeling of a human operator's control actions to navigate and to park a car. Larkin [4] has proposed a fuzzy controller for aircraft flight control where the fuzzy rules are generated by interrogating an experienced pilot and asking him a number of highly structured questions. In [5], the authors have designed an autopilot for ships by translating the steering behavior of a human controller into a fuzzy mathematical model. In [6], a fuzzy control that uses rules on a skilled human operator's experience is applied to automatic train operations. Nguyen and Widow [7] have developed a neural network controller for the truck backer upper to a loading dock problem from an arbitrary initial position by manipulating the steering. Kong and Kosko [8] have proposed a fuzzy control strategy for the same problem. In [9],

Wang has solved the same problem by generating fuzzy rules using learning algorithms. However, all the above studies do not treat the PT problem and have not analyzed the stability of the control systems.

This paper focuses on the design of a stabilizing fuzzy controller for the PT problem of vehicles using its nonlinear dynamics model. Such a dynamics model has been developed [23,24], and is used by vehicle constructors [25] in order to simulate the vehicle behavior. The vehicle nonlinear model will be approximated by a set of linear models interpolated by membership functions (Takagi–Sugeno (T–S) fuzzy model) and then a model-based fuzzy controller will be developed to stabilize the T–S fuzzy model. Based on the T–S fuzzy model of the vehicle, the outcome of the fuzzy tracking control problem is parameterized in terms of a linear matrix inequality (LMI) problem. The LMI problem can be solved very efficiently by convex optimization techniques to complete the fuzzy path tracking control design for vehicles.

The paper is organized as follows. In Section 2, the vehicle dynamics model is presented, thereafter, a kinematics model based on mobile target configuration tracking is derived for the PT problem. Section 3 is devoted to the representation of the vehicle model by a T–S fuzzy model and to the fuzzy control design for PT problem using Lyapunov's approach combined by the LMIs approach. In Section 4, simulation results are given to highlight the effectiveness of the proposed control law. Section 5 concludes the paper.

2. Problem statement

2.1. Car dynamics model

Generally, the real time application linked to the control of the vehicle use kinematics models or dynamics ones which take into consideration one or two degrees of freedom, for example lateral displacement and yaw angle [26]. In order to obtain a good accuracy on the behavior of the vehicle, we propose to use lateral and longitudinal dynamics. We consider that the vehicle moves on a plane road, dry and without disturbances such as wind, snow and rain. We do not consider the vertical displacement and pith angle. With these considerations, the final dynamics model describing the behavior of the vehicle, as introduced in [18,24] is given by

$$
\begin{aligned}
\dot{u} &= vr - fg + \frac{fk_1 - k_2}{M}u^2 + \frac{C_f}{M}\left(\frac{v}{u} + \frac{ar}{u}\right)\delta + \frac{1}{M}T, \\
\dot{v} &= -ur - \frac{C_f + C_r}{M}\frac{v}{u} + \frac{bC_r - aC_f}{M}\frac{r}{u} + \frac{C_f}{M}\delta + \frac{1}{M}T\delta, \\
\dot{r} &= -\frac{fMh}{I_z}ur + \frac{bC_r - aC_f}{T_z}\frac{v}{u} - \frac{b^2C_r - a^2C_f}{I_z}\frac{r}{u} + \frac{aC_f}{I_z}\delta + \frac{a}{I_z}T\delta, \\
\dot{\phi} &= r,
\end{aligned}
\tag{1}
$$

where (x, y) are the longitudinal and lateral displacement, respectively, in the fixed frame $R(0, \vec{i}, \vec{j})$, (u, v) are longitudinal and lateral velocities, respectively, r the yaw rate and ϕ the orientation angle of the vehicle. The control actions of the vehicle are the traction force or the braking force T and the steering angle δ of the front wheel (see Fig. 1). Constants of the system are indicated in Appendix A. We remark that this dynamics model is MIMO and highly nonlinear.

2.2. Path tracking problem

As shown in Fig. 1, the target configuration is represented by a reference vehicle with the same kinematics constraints as the real one.

Let (u, v, ϕ) and (u_r, v_r, ϕ_r) be, respectively, the longitudinal velocity, lateral velocity, yaw rate of the real and the reference vehicle.
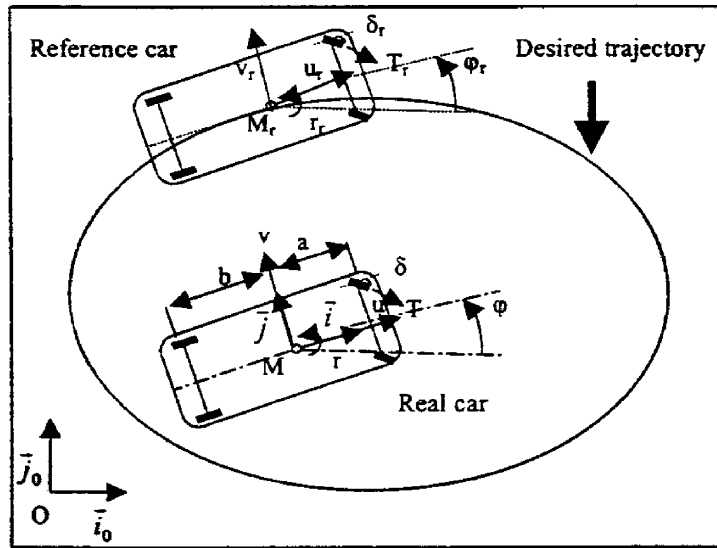
Fig. 1. Path following representation.

Our objective is to determine the control actions T and δ allowing the real vehicle to follow a trajectory defined by reference vehicle. In other words, let (x_e, y_e) are the coordinates of the position error vector $\vec{MM_r}$ in the frame $R_l(M, \vec{i}, \vec{j})$ linked to the real vehicle, and $\phi_e = \phi - \phi_r$ denote the orientation error between both vehicles. The position error vector can be written in the mobile frame R_l as follows:

$$\vec{MM_r} = x_e \vec{i} + y_e \vec{j}. \tag{2}$$

Differentiating (2) with respect to time yields

$$\frac{d\vec{MM_r}}{dt} = \dot{x}_e \vec{i} + \dot{y}_e \vec{j} + x_e \dot{\phi} \vec{j} - y_e \dot{\phi} \vec{i} = u_e \vec{i} + v_e \vec{i}, \tag{3}$$

i.e.,

$$\dot{x}_e = u_e - y_e r, \qquad \dot{y}_e = v_e - x_e r.$$

Furthermore, we have

$$\frac{d\vec{MM_r}}{dt} = \frac{d(\vec{OM_r})}{dt} - \frac{d(\vec{OM})}{dt}, \tag{4}$$

where

$$\frac{d\vec{OM_r}}{dt} = u_r \cos(\phi_e) \vec{i} - u_r \sin(\phi_e) \vec{i} + v_r \sin(\phi_e) \vec{i} + v_r \cos(\phi_e) \vec{j}, \tag{5}$$

$$\frac{d\vec{OM}}{dt} = u \vec{i} + v \vec{j}. \tag{6}$$

Substituting (3), (5) and (6) into (4), one obtains

$$\dot{x}_e = u_r \cos(\phi_e) + v_r \sin(\phi_e) - u + y_e \dot{\phi},$$
$$\dot{y}_e = -u_r \sin(\phi_e) + v_r \cos(\phi_e) - v - x_e \dot{\phi}, \tag{7}$$

i.e.,

$$u = u_r \cos(\phi_e) + v_r \sin(\phi_e) - u_e, \qquad v = -u_r \sin(\phi_e) + v_r \cos(\phi_e) - v_e.$$

Furthermore, from Fig. 1, one has

$$\dot{\phi}_e = \dot{\phi} - \dot{\phi}_r = r_e = r - r_r. \tag{8}$$

Finally, the state representation for the path tracking problem can be written as follows:

$$
\begin{aligned}
\dot{u} &= vr - fg + \frac{fk_1 - k_2}{M}u^2 + \frac{C_f}{M}\left(\frac{v}{u} + \frac{ar}{u}\right)\delta + \frac{1}{M}T, \\
\dot{v} &= -ur - \frac{C_f + C_r}{M}\frac{v}{u} + \frac{bC_r - aC_f}{M}\frac{r}{u} + \frac{C_f}{M}\delta + \frac{1}{M}T\delta, \\
\dot{r} &= -\frac{fMh}{I_z}ur + \frac{bC_r - aC_f}{T_z}\frac{v}{u} - \frac{b^2C_r + a^2C_f}{I_z}\frac{r}{u} + \frac{aC_f}{I_z}\delta + \frac{a}{I_z}T\delta, \\
\dot{x}_e &= u_r \cos(\phi_e) + v_r \sin(\phi_e) - u + y_e\dot{\phi}, \\
\dot{y}_e &= -u_r \sin(\phi_e) + v_r \cos(\phi_e) - v - x_e\dot{\phi}, \\
\dot{\phi}_e &= \dot{\phi} - \dot{\phi}_r.
\end{aligned}
\tag{9}
$$

Our control objective is to make the vehicle follow a desired trajectory, that is

$$u \to u_r, \qquad v \to v_r, \qquad x_e \to 0, \qquad y_e \to 0, \qquad \phi_e \to 0.$$

3. Analysis and design of fuzzy control system

3.1. T–S fuzzy model of vehicle

As in [28], we propose to use T–S fuzzy control for the nonlinear system trajectory tracking problem described by Eq. (9).

Using this technique, the T–S fuzzy model of a vehicle is easily obtained by linearization near different operation points (X_{ei}, U_{ei}).

The nonlinear model given in (9) has the following form:

$$\dot{X} = F(X, U, t), \tag{10}$$

where $[F_1, \ldots, F_6]$ is a six-dimensional vector function of the state vector $X = [u, v, r, x_e, y_e, \phi_e]$ and control vector $U = [T, \delta]$. The functions F_i are continuous and continuously differentiable in their arguments. At the equilibrium

$$F(X_e, U_e, t) = 0.$$

After linearization of nonlinear model described in Eq. (10) for a specific equilibrium point using Taylor series, we obtain a T–S fuzzy model with the following form:

$$L^i : \text{ if } X \text{ is } \sim (X_{ei}, U_{ei}), \text{ then } \dot{\bar{X}} = A_i\bar{X} + B_i\bar{U},$$

where

$$
A_i = \begin{pmatrix}
\dfrac{2u_{ei}f(k_1-k_2)}{M} - \dfrac{C_f(v_{ei}+ar_{ei})\delta_{ei}}{Mu_{ei}^2} & r_{ei} + \dfrac{C_f\delta_{ei}}{Mu_{ei}} & v_{ei} + \dfrac{aC_f\delta_{ei}}{Mu_{ei}} & 0 & 0 & 0 \\[2ex]
\dfrac{C_f+C_r}{Mu_{ei}^2}v_{ei} - \dfrac{bC_r-aC_f}{Mu_{ei}^2}r_{ei} - r_{ei} & -\dfrac{C_f+C_r}{Mu_{ei}} & \dfrac{bC_r-aC_f}{Mu_{ei}} - u_{ei} & 0 & 0 & 0 \\[2ex]
-\dfrac{fMh}{I_z}r_{ei} + \dfrac{b^2C_r+a^2C_f}{I_zu_{ei}^2} - \dfrac{bC_r-aC_r}{I_zu_{ei}^2} & \dfrac{bC_r-aC_r}{I_zu_{ei}} & -\dfrac{fMh}{I_z}u_{ei} - \dfrac{b^2C_r+a^2C_f}{I_zu_{ei}} & 0 & 0 & 0 \\[2ex]
-1 & 0 & 0 & 0 & r_{ei} & v_{ei} \\[1.5ex]
0 & -1 & 0 & -r_{ei} & 0 & -u_{ei} \\[1.5ex]
0 & 0 & 1 & 0 & 0 & 0
\end{pmatrix},
$$

$$
B_i = \begin{pmatrix}
\dfrac{1}{M} & \dfrac{C_f(v_{ei}+ar_{ei})}{Mu_{ei}} \\[2ex]
\dfrac{\delta_{ei}}{M} & \dfrac{T_{ei}+C_f}{M} \\[2ex]
\dfrac{a\delta_{ei}}{I_z} & a\dfrac{T_{ei}+C_f}{I_z} \\[2ex]
0 & 0 \\[1ex]
0 & 0 \\[1ex]
0 & 0
\end{pmatrix}.
$$

The fuzzy rule L^i represents the ith linearized system about the operating point (X_{ei}, U_{ei}), where

$$
\bar{X} = X - X_{ei}, \qquad \bar{U} = U - U_{ei}, \qquad A_i = \left.\frac{\partial F}{\partial X}\right|_{(X_{ei}, U_{ei})}, \qquad B_i = \left.\frac{\partial F}{\partial U}\right|_{(X_{ei}, U_{ei})},
$$

L^i $(i = 1, \ldots, n)$ denotes the ith implication. (A_i, B_i) is the ith local model of the fuzzy system. Let W_i be the membership function of the inferred fuzzy set corresponding to the operating regime (X_{ei}, U_{ei}). The final state of the system is inferred by taking the weighted average of all local models:

$$
\dot{\bar{X}} = \frac{\sum_{i=1}^n W_i(A_i\bar{X} + B_i\bar{U})}{\sum_{i=1}^n W_i}. \tag{11}
$$

3.2. Fuzzy controller design

At this stage, we present the used T–S fuzzy controller scheme for nonlinear system trajectory tracking. We consider a finite number of operating regime (X_{ei}, U_{ei}). In each one, the system is characterized by local linear models. We suppose that all states in the vector $X = [u, v, r, x_e, y_e, \phi_e]$ are measured. For each local model (A_i, B_i), we design a local state feedback controllers having the following structure:

$$
\bar{U} = -K_i^f(X - X_r), \tag{12}
$$

where X_r is the reference model state and K_i^f is any matrix such that $(A_i - B_iK_i^f)$ is Hurwitz. The global controller is inferred by calculating the weighted average of all local controllers. The membership functions are used as smooth interpolations.

The structure of the fuzzy controller is

$$R^i : \text{if } X \text{ is } (X_{\mathrm{e}i}, U_{\mathrm{e}i}), \text{ then } \bar{U} = -K_i^{\mathrm{f}} \bar{X}.$$

The final output of fuzzy control is given as

$$\bar{U} = -\frac{\sum_{i=1}^{n} W_i K_i^{\mathrm{f}} \bar{X}}{\sum_{i=1}^{n} W_i}. \tag{13}$$

3.3. Stability analysis

It is well known that even if the local controllers stabilize the corresponding local models, the global stability of the closed loop system is not guaranteed.

Next, we will find a sufficient condition with guaranteeing a global stability. Now the global closed loop fuzzy system is obtained:

$$\dot{X} = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} W_i W_j [A_i - B_i K_i^{\mathrm{f}}] X}{\sum_{j=1}^{n} \sum_{i=1}^{n} W_j W_i}. \tag{14}$$

The sufficient conditions for exponential stability of (14) are well known [13–15].

Theorem 1. *The equilibrium of fuzzy system* (14) *is asymptotically stable if there exists a common positive definite matrix P such that*

$$(A_i - B_i K_i^{\mathrm{f}})^{\mathrm{T}} P + P(A_i - B_i K_i^{\mathrm{f}}) < 0 \quad for\, i = 1, \ldots, n, \tag{15}$$

$$G_{ij}^{\mathrm{T}} P + P G_{ij} < 0 \quad for\, i < j < n, \tag{16}$$

where

$$G_{ij} = \tfrac{1}{2}([A_i - B_i K_j^{\mathrm{f}}] + [A_j - B_j K_i^{\mathrm{f}}]).$$

This theorem reduces to the Lyapunov stability theorem for continuous time linear systems when $n = 1$. The control design problem is to select K_i^{f} ($i = 1, \ldots, n$) such that conditions (15) and (16) in Theorem 1 are satisfied.

To check the stability of the fuzzy control system, it has long been considered difficult to find a common positive definite matrix P satisfying conditions (15) and (16). In [13], a procedure to construct a common P is given for second order fuzzy systems. We pointed out in [14] that the problem of finding a common matrix P can be solved numerically by convex programming algorithms involving LMIs [21,27]. To do this, a very important observation is that the stability condition of Theorem 1 is expressed in LMIs [15]. To check the stability, we need to find P satisfying the LMI conditions

$$(A_i - B_i K_i^{\mathrm{f}})^{\mathrm{T}} P + P(A_i - B_i K_i^{\mathrm{f}}) < 0 \quad \text{for } i = 1, 2, \ldots, n, \qquad G_{ij}^{\mathrm{T}} P + P G_{ij} < 0 \quad \text{for } i < j < n.$$

4. Simulation results

For the path tracking maneuver, we choose the operating points which check the following equations:

$$T_{\mathrm{e}} + M v_{\mathrm{e}} r_{\mathrm{e}} - M f g + C_{\mathrm{f}} \delta_{\mathrm{e}} \frac{v_{\mathrm{e}} + a r_{\mathrm{e}}}{u_{\mathrm{e}}} + u_{\mathrm{e}}^2 (f k_1 - k_2) = 0,$$

$$T_{\mathrm{e}} \delta_{\mathrm{e}} - M u_{\mathrm{e}} r_{\mathrm{e}} + C_{\mathrm{f}} \delta_{\mathrm{e}} - (C_{\mathrm{f}} + C_{\mathrm{r}}) \frac{v_{\mathrm{e}}}{u_{\mathrm{e}}} + (C_{\mathrm{r}} - a C_{\mathrm{f}}) \frac{r_{\mathrm{e}}}{u_{\mathrm{e}}} = 0, \tag{17}$$

$$a T_{\mathrm{e}} \delta_{\mathrm{e}} - M f h u_{\mathrm{e}} r_{\mathrm{e}} + a C_{\mathrm{f}} \delta_{\mathrm{e}} - (a C_{\mathrm{e}} - b C_{\mathrm{r}}) \frac{v_{\mathrm{e}}}{u_{\mathrm{e}}} - (b^2 C_{\mathrm{r}} + a^2 C_{\mathrm{f}}) \frac{r_{\mathrm{e}}}{u_{\mathrm{e}}} = 0.$$

Three operating points are chosen:

$$(u_{e1}, v_{e1}, r_{e1}, T_{e1}, \delta_{e1}) = (20\,\text{m/s}, 0\,\text{m/s}, 0\,\text{rad/s}, 454.33\,\text{N}, 0°),$$
$$(u_{e2}, v_{e2}, r_{e2}, T_{e2}, \delta_{e2}) = (30\,\text{m/s}, -3.2\,\text{m/s}, 0.8\,\text{rad/s}, 5438.3\,\text{N}, 5°),$$
$$(u_{e3}, v_{e3}, r_{e3}, T_{e3}, \delta_{e3}) = (40\,\text{m/s}, -8\,\text{m/s}, 1\,\text{rad/s}, 15\,975\,\text{N}, 5°).$$

We consider the following gains:

$$K_1^f = \begin{bmatrix} 2.8466 \times 10^4 & -9.5333 \times 10^3 & 5.9163 \times 10^4 & -2.4935 \times 10^5 & 3.2292 \times 10^5 & 1.0677 \times 10^5 \\ -3.0888 \times 10^{-3} & 0.13649 & 0.087483 & -0.087662 & -0.88671 & 6.8637 \end{bmatrix},$$

$$K_2^f = \begin{bmatrix} 3.6311 \times 10^4 & 3.0077 \times 10^4 & 5.8368 \times 10^5 & -3.1659 \times 10^5 & 2.9973 \times 10^2 & 3.2963 \times 10^6 \\ -1.9610 \times 10^{-2} & 1.3031 \times 10^{-1} & -2.0189 \times 10^{-1} & 4.4963 \times 10^{-2} & -8.6899 \times 10^{-1} & 5.3208 \end{bmatrix},$$

$$K_3^f = \begin{bmatrix} 3.1642 \times 10^4 & -2.2318 \times 10^5 & 2.4846 \times 10^5 & -1.9236 \times 10^5 & 1.5170 \times 10^6 & -8.6682 \times 10^6 \\ -1.3971 \times 10^{-2} & 2.8546 \times 10^{-1} & -1.5647 \times 10^{-2} & -1.6701 \times 10^{-1} & -1.8193 & 1.2974 \times 10^1 \end{bmatrix}.$$

Fig. 2 gives the membership functions of the longitudinal velocity corresponding to regime (X_{ei}, U_{ei}).

The fuzzy model of the vehicle designed from three rules is stable if

$$S_{11} = G_{11}^T P + P G_{11} < 0, \tag{18}$$

$$S_{22} = G_{22}^T P + P G_{22} < 0, \tag{19}$$

$$S_{33} = G_{33}^T P + P G_{33} < 0, \tag{20}$$

$$S_{12} = G_{12}^T P + P G_{12} < 0, \tag{21}$$

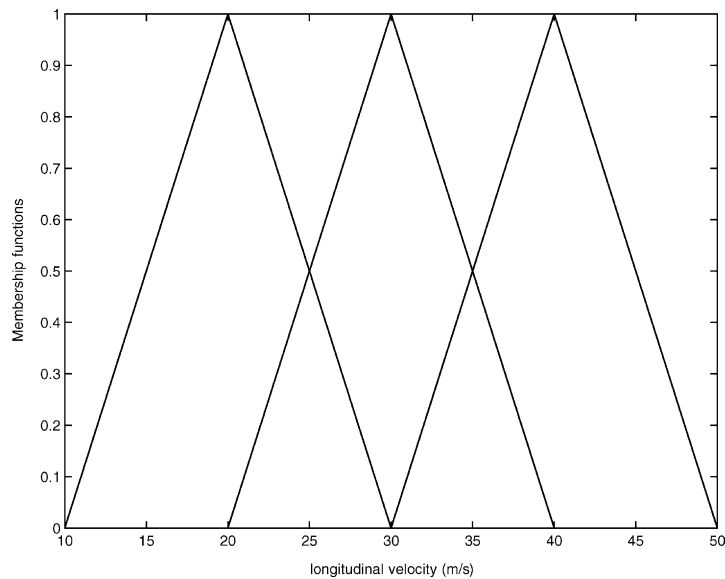$$S_{13} = G_{13}^T P + P G_{13} < 0, \tag{22}$$
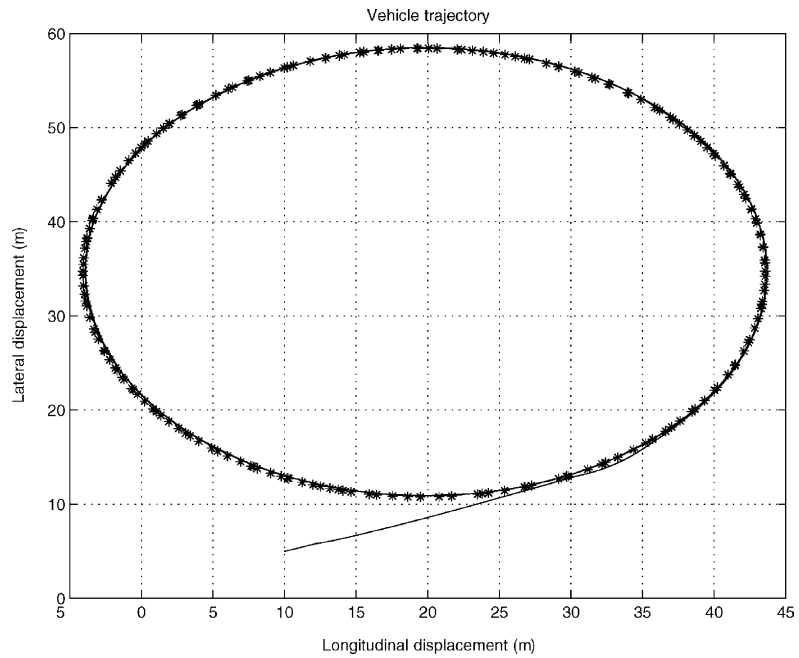

Fig. 2. Membership functions.

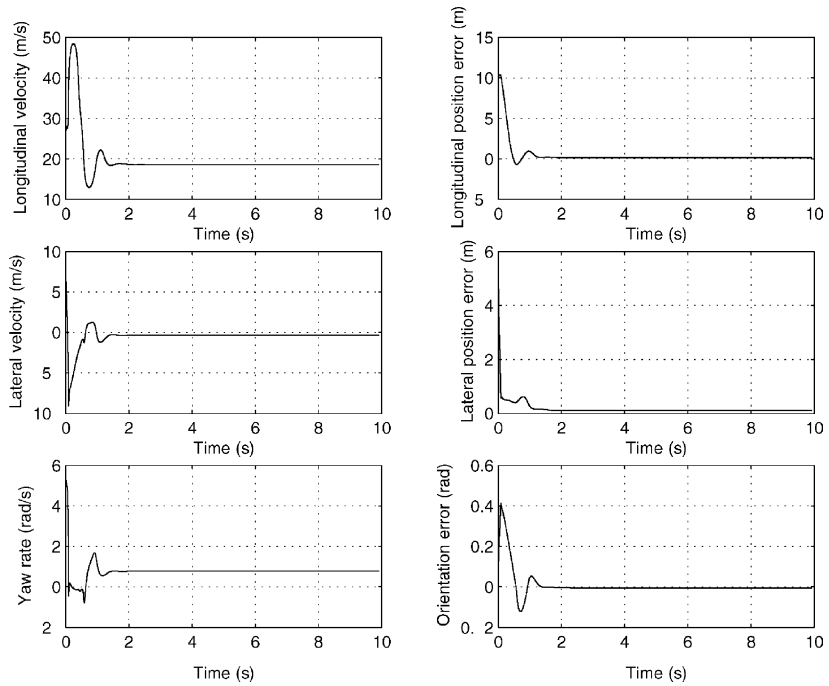Fig. 3. Motion vehicle to the desired trajectory.

Fig. 4. State variable evolution.

$$S_{23} = G_{23}^{\mathrm{T}} P + P G_{23} < 0 \tag{23}$$

for a common positive matrix P.

Using the LMI approach, we obtain

$$P = \begin{bmatrix} 2.4559 \times 10^{-6} & 9.4152 \times 10^{-7} & -1.8604 \times 10^{-8} & 2.6138 \times 10^{-7} & 1.1279 \times 10^{-7} & 3.3387 \times 10^{-1} \\ 9.4152 \times 10^{-7} & 5.4117 \times 10^{-7} & 3.4499 \times 10^{-9} & 1.1973 \times 10^{-7} & 6.1704 \times 10^{-8} & -8.5123 \times 10^{-10} \\ -1.8604 \times 10^{-8} & 3.4499 \times 10^{-9} & 2.0344 \times 10^{-9} & -2.0723 \times 10^{-9} & -3.0940 \times 10^{-10} & -1.5350 \times 10^{-1} \\ 2.6138 \times 10^{-7} & 1.1973 \times 10^{-7} & -2.0723 \times 10^{-9} & 3.5686 \times 10^{-8} & 1.5424 \times 10^{-8} & 2.6756 \times 10^{-10} \\ 1.1279 \times 10^{-7} & 6.1704 \times 10^{-8} & -3.0940 \times 10^{-10} & 1.5424 \times 10^{-8} & 7.8649 \times 10^{-9} & 5.0227 \times 10^{-11} \\ 3.3387 \times 10^{-10} & -8.5123 \times 10^{-10} & -1.5350 \times 10^{-10} & 2.6756 \times 10^{-10} & 5.0227 \times 10^{-11} & 3.2539 \times 10^{-11} \end{bmatrix} > 0.$$

It can be easily shown that the stability conditions (18)–(23) are satisfied.

To highlight the effectiveness of the proposed control algorithm, we present the simulation results for a path tracking problem of the vehicle. One arbitrarily chosen initial state $[u_0, v_0, r_0, x_0, y_0, \phi_0] = [25\,\mathrm{m/s}, -0.8\,\mathrm{m/s}, 0.2\,\mathrm{rad/s}, 10\,\mathrm{m}, 5\,\mathrm{m}, 0.1\,\mathrm{rad}]$ and the desired trajectory is defined by $(u_r = 18.5\,\mathrm{m/s}, v_r = -0.34\,\mathrm{m/s}, r_r = 0.78\,\mathrm{rad/s})$. Fig. 3 shows that the vehicle starting from initial state follows the desired trajectory rapidly. Fig. 4 shows the convergence of the lateral velocity, longitudinal velocity, yaw rate variables towards their desired velocity and the tracking errors of longitudinal displacement, lateral displacement and the orientation between two vehicles. We see that this fuzzy controller successfully drives the vehicle to the desired trajectory starting from an arbitrary initial state.

5. Conclusion

In this paper, we have presented a T–S fuzzy scheme for trajectory tracking of vehicle dynamics. A nonlinear behavior of vehicle has been presented by a T–S fuzzy model. Based on this T–S fuzzy model, a fuzzy controller has been developed. The stability of the closed loop nonlinear systems has been analyzed using Lyapunov's method combined with an LMI approach. A simulation result is given to illustrate the designed procedure and tracking performance of the proposed algorithm.

Appendix A

Symbol	Name	Value
a (mm)	Distance, c.g. to front axle	1050
b (mm)	Distance, c.g. to rear axle	1630
h (mm)	c.g. height	530
M (kg)	Total mass	1480
f	Nominal friction coefficient	0.02
I_z (kg m^2)	Moment of inertia	2350
g (m/s^2)	Acceleration due to gravity	8.81
C_f (N/rad)	Front roll stiffness	135000
C_r (N/rad)	Rear roll stiffness	95000
k_1 (N/s m)	Portance parameter	0.005
k_2 (N/s m)	Drag parameter	0.41
f_b	Distribution coefficient, front/rear	0.6

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